From Chapter 1:

1. 4.8

2. 6.4. Also, recall the definition, from the text, of the inverse d.f.:

\[ F^{-1}(t) = \min \{ x | F(x) \geq t \} . \]

Most authors use inf rather than min. Show that min is correct, in that the inf is in fact attained.

3. 6.9

From Chapter 2:

4. Show that \( Y_n \overset{q.m.}{\to} c \implies E[Y_n] \to c \) as long as the expectations exist.

5. Prove that if \( Y_n \overset{L}{\to} Y \sim F \) then \( P(Y_n < c) \to P(Y < c) \) if \( F \) is continuous at \( c \). What if \( F \) is not continuous at \( c \)? Illustrate your answer with a concrete example.

6. Prove the following special case of Slutsky’s Theorem: If \( X_n \overset{L}{\to} X \sim F \) and \( B_n \overset{p.r.}{\to} b \), then \( X_n + B_n \overset{L}{\to} X + b \). (No credit if your “proof” uses Slutsky’s Theorem itself!)

7. 1.8 Then use (i) of this question, together with the characterization of convergence in law in terms of the convergence of certain expectations, to give an alternate proof (i.e. without using Slutsky’s Theorem) of the result that if \( X_n \overset{p.r.}{\to} a \) (constant) then \( X_n \overset{L}{\to} a \).

8. 2.11

9. 3.15

10. 4.12

11. 5.2

12. 8.4

...over
13. This is an example of the statement that the characteristic function uniquely determines the distribution function. Let $X$ be a discrete random variable with $P(X = n) = p_n$ for $n = 1, 2, ..., N$. Let $\psi(t) = E [e^{itX}]$ be the c.f. Show that

$$
\sum_{k=1}^{N} \psi(tk)e^{-itkn} = Np_n,
$$

if $t = 2\pi/N$.

14. There is some satisfaction in obtaining a sophisticated mathematical result as a consequence of a seemingly much simpler probabilistic result. Obtain Stirling’s approximation (1.1.26 in Chapter 1) in the following manner. Let $Y_n$ be the sum of $n$ i.i.d. $\mathcal{P}(1)$ r.v.s. Then $\sqrt{n}P(Y_n = n)$ can be gotten from the exact distribution of $Y_n$. Equate this to its asymptotic expression, with error $o(1)$, obtained by writing it as

$$
\sqrt{n}P \left( -\frac{1}{2\sqrt{n}} < \frac{Y_n - n}{\sqrt{n}} \leq \frac{1}{2\sqrt{n}} \right)
$$

and then using an Edgeworth expansion of the d.f. of $(Y_n - n) / \sqrt{n}$.